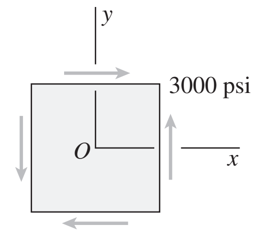
Problem 7.4-7 An element in *pure shear* is subjected to stresses $\tau_{xy} = 3000$ psi, as shown in the figure.

Using Mohr's circle, determine (a) the stresses acting on an element oriented at a counterclockwise angle $\theta = 70°$ from the x axis and (b) the principal stresses. Show all results on sketches of properly oriented elements.



Solution 7.4-7 **Pure shear**

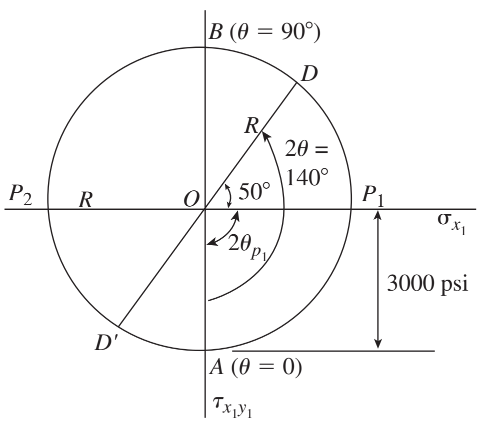
$\sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xy} = 3000$ psi

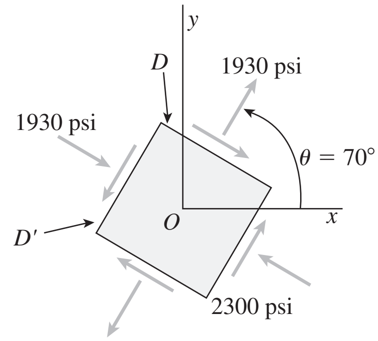
(a) Element at $\theta = 70°$

(All stresses in psi)

$2\theta = 140° \qquad \theta = 70° \qquad R = 3000$ psi

Origin O is at center of circle.





Point D: $\sigma_{x_1} = R\cos 50° = 1928$ psi

$\tau_{x_1y_1} = -R\sin 50° = -2298$ psi

Point D': $\sigma_{x_1} = -R\cos 50° = -1928$ psi

$\tau_{x_1y_1} = R\sin 50° = 2298$ psi

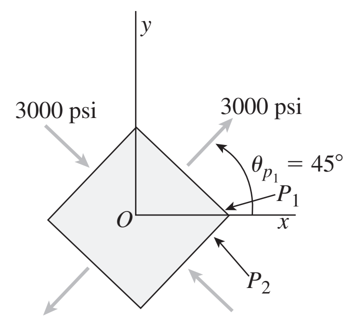
(b) Principal stresses

Point P_1: $2\theta_{p_1} = 90° \qquad \theta_{p_1} = 45°$

$\sigma_1 = R = 3000$ psi

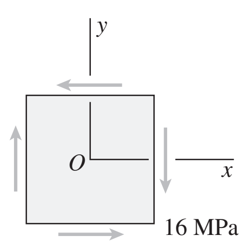
Point P_2: $2\theta_{p_2} = -90° \qquad \theta_{p_2} = -45°$

$\sigma_2 = -R = -3000$ psi



Problem 7.4-8 An element in *pure shear* is subjected to stresses $\tau_{xy} = -16$ MPa, as shown in the figure.

Using Mohr's circle, determine (a) the stresses acting on an element oriented at a counterclockwise angle $\theta = 20°$ from the x axis and (b) the principal stresses. Show all results on sketches of properly oriented elements.

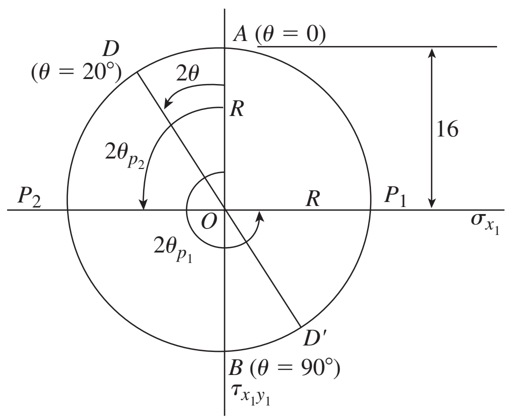


Solution 7.4-8 **Pure shear**

$\sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xy} = -16$ MPa

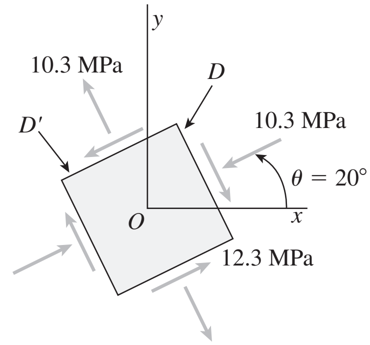
(a) ELEMENT AT $\theta = 20°$

(All stresses in MPa)
$2\theta = 40° \quad \theta = 20° \quad R = 16$ MPa
Origin O is at center of circle.



Point D: $\sigma_{x_1} = -R \sin 2\theta = -10.28$ MPa
$\tau_{x_1y_1} = -R \cos 2\theta = -12.26$ MPa

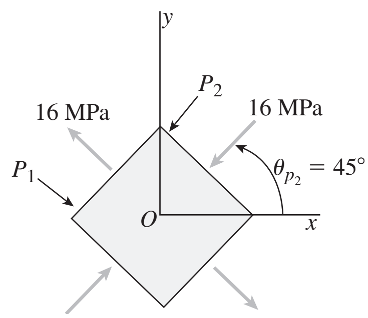
Point D': $\sigma_{x_1} = R \sin 2\theta = 10.28$ MPa
$\tau_{x_1y_1} = R \cos 2\theta = 12.26$ MPa



(b) PRINCIPAL STRESSES

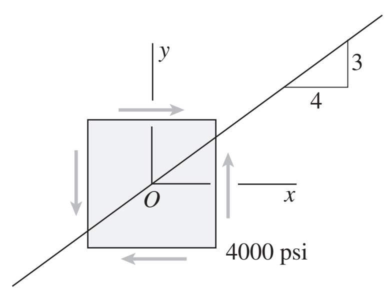
Point P_1: $2\theta_{p_1} = 270° \quad \theta_{p_1} = 135°$
$\sigma_1 = R = 16$ MPa

Point P_2: $2\theta_{p_2} = 90° \quad \theta_{p_2} = 45°$
$\sigma_2 = -R = -16$ MPa



Problem 7.4-9 An element in *pure shear* is subjected to stresses $\tau_{xy} = 4000$ psi, as shown in the figure.

Using Mohr's circle, determine (a) the stresses acting on an element oriented at a slope of 3 on 4 (see figure) and (b) the principal stresses. Show all results on sketches of properly oriented elements.

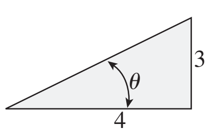


Solution 7.4-9 **Pure shear**

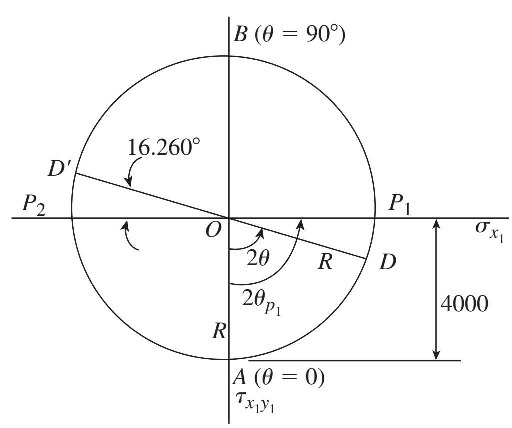
$\sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xy} = 4000$ psi

(a) ELEMENT AT A SLOPE OF 3 ON 4

(All stresses in psi) $\quad \theta = \arctan \dfrac{3}{4} = 36.870°$

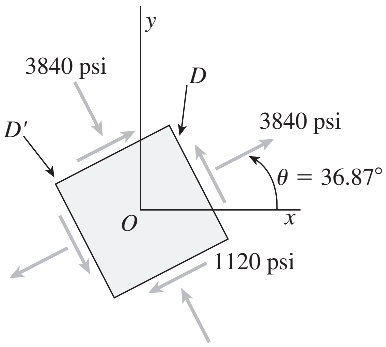


$2\theta = 73.740° \quad \theta = 36.870°$
$R = 4000$ psi
Origin O is at center of circle.



Point D: $\sigma_{x_1} = R\cos 16.260° = 3840$ psi
$\tau_{x_1y_1} = R\sin 16.260° = 1120$ psi

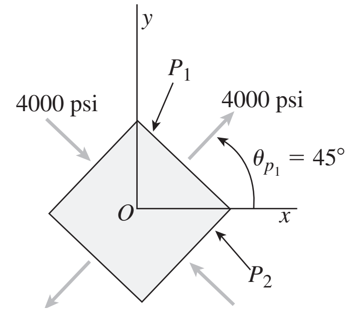
Point D': $\sigma_{x_1} = -R\cos 16.260° = -3840$ psi
$\tau_{x_1y_1} = -R\sin 16.260° = -1120$ psi



(b) PRINCIPAL STRESSES

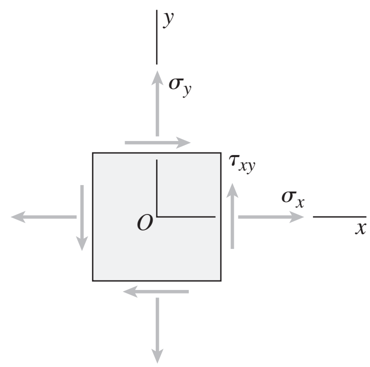
Point P_1: $2\theta_{p_1} = 90°$ $\theta_{p_1} = 45°$
$\sigma_1 = R = 4000$ psi

Point P_2: $2\theta_{p_2} = -90°$ $\theta_{p_2} = -45°$
$\sigma_2 = -R = -4000$ psi



Problems 7.4-10 through 7.4-15 An element in *plane stress* is subjected to stresses σ_x, σ_y, and τ_{xy} (see figure).

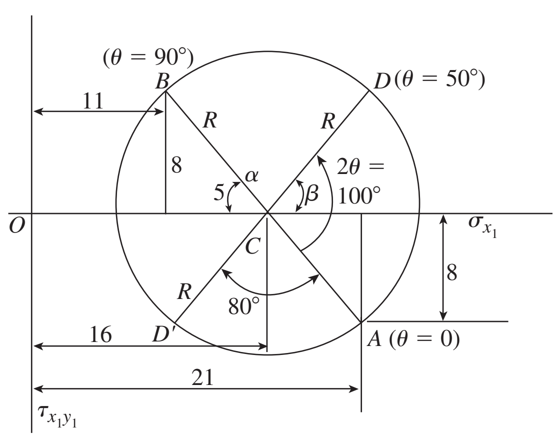
Using Mohr's circle, determine the stresses acting on an element oriented at an angle θ from the x axis. Show these stresses on a sketch of an element oriented at the angle θ. (*Note:* The angle θ is positive when counterclockwise and negative when clockwise.)



Data for 7.4-10 $\sigma_x = 21$ MPa, $\sigma_y = 11$ MPa, $\tau_{xy} = 8$ MPa, $\theta = 50°$

Solution 7.4-10 Plane stress (angle θ)

$\sigma_x = 21$ MPa $\sigma_y = 11$ MPa
$\tau_{xy} = 8$ MPa $\theta = 50°$
(All stresses in MPa)



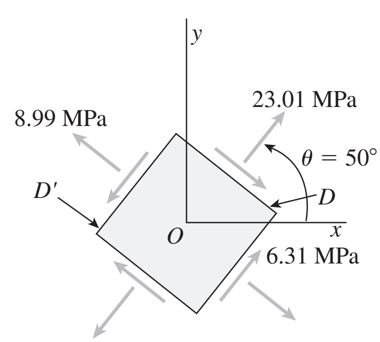
$R = \sqrt{(5)^2 + (8)^2} = 9.4340$ MPa

$\alpha = \arctan\frac{8}{5} = 57.99°$

$\beta = 2\theta - \alpha = 100° - \alpha = 42.01°$

Point D ($\theta = 50°$):
$\sigma_{x_1} = 16 + R\cos\beta = 23.01$ MPa
$\tau_{x_1y_1} = -R\sin\beta = -6.31$ MPa

Point D' ($\theta = -40°$):
$\sigma_{x_1} = 16 - R\cos\beta = 8.99$ MPa
$\tau_{x_1y_1} = R\sin\beta = 6.31$ MPa



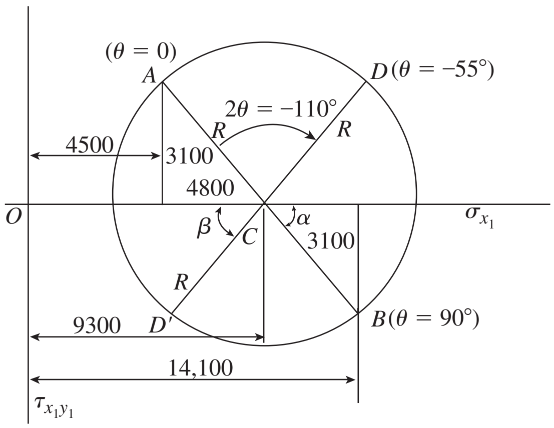
Data for 7.4-11 $\sigma_x = 4500$ psi, $\sigma_y = 14{,}100$ psi, $\tau_{xy} = -3100$ psi, $\theta = -55°$

Solution 7.4-11 Plane stress (angle θ)

$\sigma_x = 4500$ psi $\quad \sigma_y = 14{,}100$ psi

$\tau_{xy} = -3100$ psi $\quad \theta = -55°$

(All stresses in psi)



$$R = \sqrt{(4800)^2 + (3100)^2} = 5714 \text{ psi}$$

$$\alpha = \arctan \frac{3100}{4800} = 32.86°$$

$\beta = 180° - 110° - \alpha = 37.14°$

Point D ($\theta = -55°$):

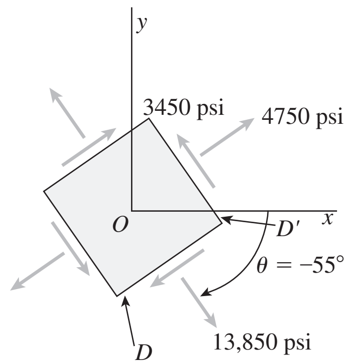
$$\sigma_{x_1} = 9300 + R\cos\beta = 13{,}850 \text{ psi}$$

$$\tau_{x_1y_1} = -R\sin\beta = -3450 \text{ psi}$$

Point D' ($\theta = 35°$):

$$\sigma_{x_1} = 9300 - R\cos\beta = 4750 \text{ psi}$$

$$\tau_{x_1y_1} = R\sin\beta = 3450 \text{ psi}$$



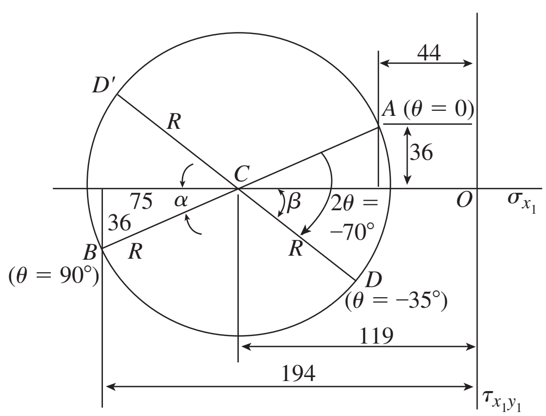
Data for 7.4-12 $\sigma_x = -44$ MPa, $\sigma_y = -194$ MPa, $\tau_{xy} = -36$ MPa, $\theta = -35°$

Solution 7.4-12 Plane stress (angle θ)

$\sigma_x = -44$ MPa $\quad \sigma_y = -194$ MPa

$\tau_{xy} = -36$ MPa $\quad \theta = -35°$

(All stresses in MPa)



$$R = \sqrt{(75)^2 + (36)^2} = 83.19 \text{ MPa}$$

$$\alpha = \arctan \frac{36}{75} = 25.64°$$

$\beta = 70° - \alpha = 44.36°$

Point D ($\theta = -35°$):

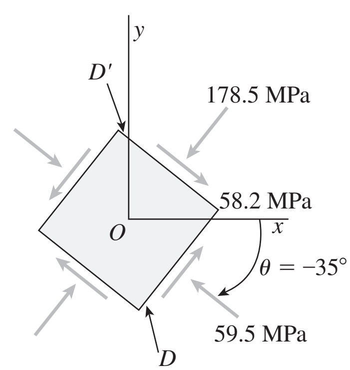
$$\sigma_{x_1} = -119 + R\cos\beta = -59.5 \text{ MPa}$$

$$\tau_{x_1y_1} = R\sin\beta = 58.2 \text{ MPa}$$

Point D' ($\theta = 55°$):

$$\sigma_{x_1} = -119 - R\cos\beta = -178.5 \text{ MPa}$$

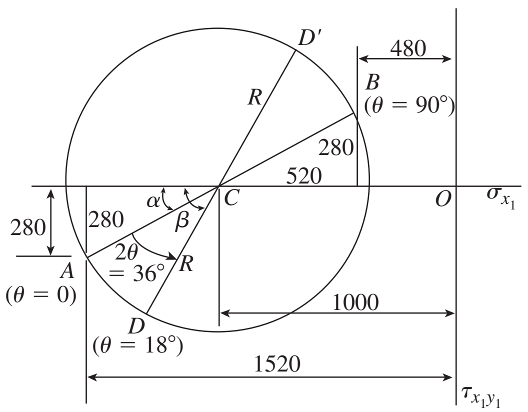
$$\tau_{x_1y_1} = -R\sin\beta = -58.2 \text{ MPa}$$



Data for 7.4-13 $\sigma_x = -1520$ psi, $\sigma_y = -480$ psi, $\tau_{xy} = 280$ psi, $\theta = 18°$

Solution 7.4-13 Plane stress (angle θ)

$\sigma_x = -1520$ psi $\quad \sigma_y = -480$ psi
$\tau_{xy} = 280$ psi $\quad \theta = 18°$
(All stresses in psi)



$$R = \sqrt{(520)^2 + (280)^2} = 590.6 \text{ psi}$$

$$\alpha = \arctan \frac{280}{520} = 28.30°$$

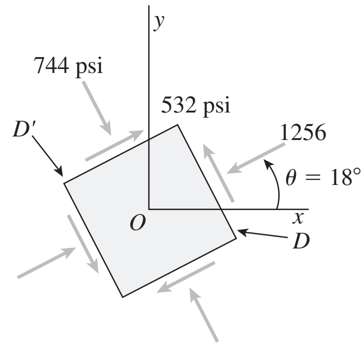
$\beta = \alpha + 36° = 64.30°$

Point D ($\theta = 18°$):

$$\sigma_{x_1} = -1000 - R\cos\beta = -1256 \text{ psi}$$
$$\tau_{x_1y_1} = R\sin\beta = 532 \text{ psi}$$

Point D' ($\theta = 108°$):

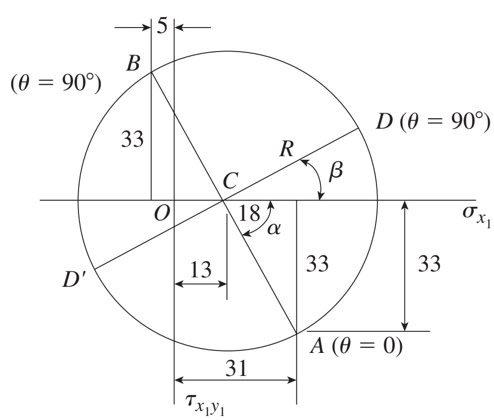
$$\sigma_{x_1} = -1000 + R\cos\beta = -744 \text{ psi}$$
$$\tau_{x_1y_1} = -R\sin\beta = -532 \text{ psi}$$



Data for 7.4-14 $\sigma_x = 31$ MPa, $\sigma_y = -5$ MPa, $\tau_{xy} = 33$ MPa, $\theta = 45°$

Solution 7.4-14 Plane stress (angle θ)

$\sigma_x = 31$ MPa $\quad \sigma_y = -5$ MPa
$\tau_{xy} = 33$ MPa $\quad \theta = 45°$
(All stresses in MPa)



$$R = \sqrt{(18)^2 + (33)^2} = 37.590 \text{ MPa}$$

$$\alpha = \arctan \frac{33}{18} = 61.390°$$

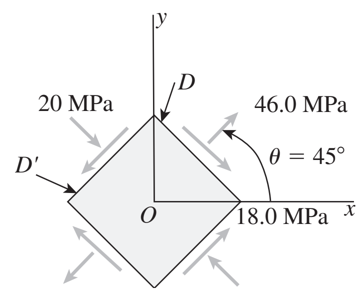
$$\beta = 90° - \alpha = 28.610°$$

Point D ($\theta = 45°$):

$$\sigma_{x_1} = 13 + R\cos\beta = 46.0 \text{ MPa}$$
$$\tau_{x_1y_1} = -R\sin\beta = -18.0 \text{ MPa}$$

Point D' ($\theta = 135°$):

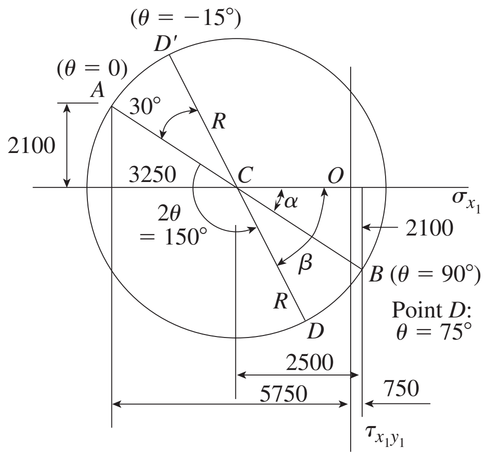
$$\sigma_{x_1} = 13 - R\cos\beta = -20.0 \text{ MPa}$$
$$\tau_{x_1y_1} = R\sin\beta = 18.0 \text{ MPa}$$



Data for 7.4-15 $\sigma_x = -5750$ psi, $\sigma_y = 750$ psi, $\tau_{xy} = -2100$ psi, $\theta = 75°$

Solution 7.4-15 **Plane stress (angle θ))**

$\sigma_x = -5750$ psi $\quad \sigma_y = 750$ psi
$\tau_{xy} = -2100$ psi $\quad \theta = 75°$
(All stresses in psi)



$$R = \sqrt{(3250)^2 + (2100)^2} = 3869 \text{ psi}$$

$$\alpha = \arctan \frac{2100}{3250} = 32.87°$$

$$\beta = \alpha + 30° = 62.87°$$

Point D ($\theta = 75°$):

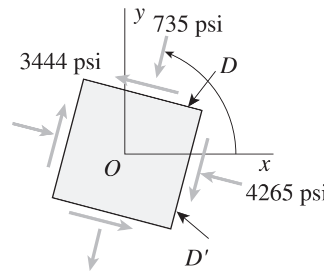
$$\sigma_{x_1} = -2500 + R \cos \beta = -735 \text{ psi}$$

$$\tau_{x_1 y_1} = R \sin \beta = 3444 \text{ psi}$$

Point D'($\theta = -15°$):

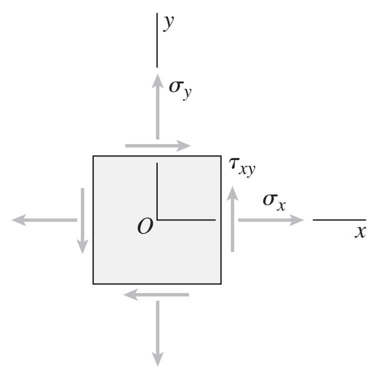
$$\sigma_{x_1} = -2500 - R \cos \beta = -4265 \text{ psi}$$

$$\tau_{x_1 y_1} = -R \sin \beta = -3444 \text{ psi}$$



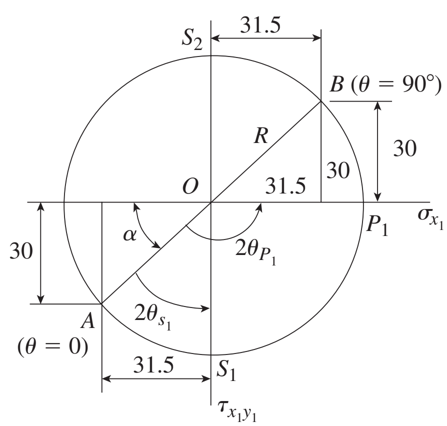
Problems 7.4-16 through 7.4-23 An element in *plane stress* is subjected to stresses σ_x, σ_y, and τ_{xy} (see figure).

Using Mohr's circle, determine (a) the principal stresses and (b) the maximum shear stresses and associated normal stresses. Show all results on sketches of properly oriented elements.



Data for 7.4-16 $\sigma_x = -31.5$ MPa, $\sigma_y = 31.5$ MPa, $\tau_{xy} = 30$ MPa

Solution 7.4-16 **Principal stresses**



$\sigma_x = -31.5$ MPa $\quad \sigma_y = 31.5$ MPa
$\tau_{xy} = 30$ MPa
(All stresses in MPa)

$$R = \sqrt{(31.5)^2 + (30.0)^2} = 43.5 \text{ MPa}$$

$$\alpha = \arctan \frac{30}{31.5} = 43.60°$$

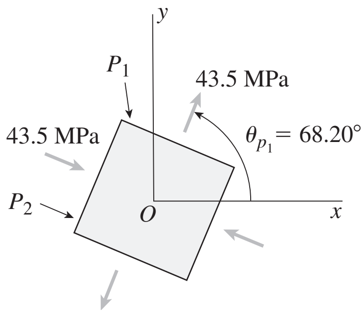
(a) PRINCIPAL STRESSES

$2\theta_{p_1} = 180° - \alpha = 136.40° \quad \theta_{p_1} = 68.20°$

$2\theta_{p_2} = -\alpha = -43.60° \quad \theta_{p_2} = -21.80°$

Point P_1: $\sigma_1 = R = 43.5$ MPa

Point P_2: $\sigma_2 = -R = -43.5$ MPa



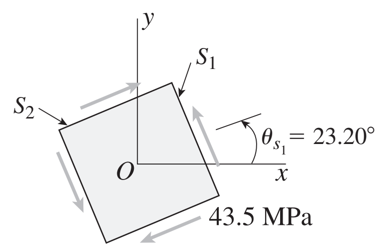
(b) MAXIMUM SHEAR STRESSES

$2\theta_{s_1} = 90° - \alpha = 46.40° \quad \theta_{s_1} = 23.20°$

$2\theta_{s_2} = 2\theta_{s_1} + 180° = 226.40° \quad \theta_{s_2} = 113.20°$

Point S_1: $\sigma_{aver} = 0 \quad \tau_{max} = R = 43.5$ MPa

Point S_2: $\sigma_{aver} = 0 \quad \tau_{min} = -43.5$ MPa

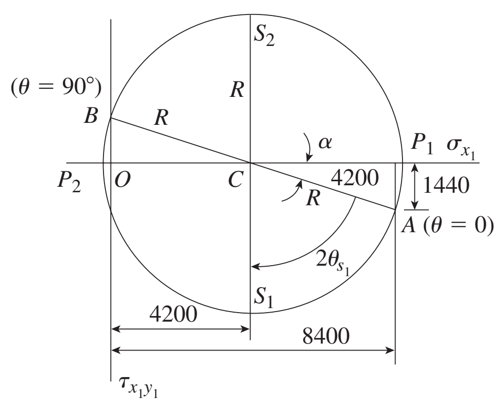


Data for 7.4-17 $\sigma_x = 8400$ psi, $\sigma_y = 0$, $\tau_{xy} = 1440$ psi

Solution 7.4-17 Principal stresses

$\sigma_x = 8400 \text{ psi} \quad \sigma_y = 0 \quad \tau_{xy} = 1440 \text{ psi}$

(All stresses in psi)



$R = \sqrt{(4200)^2 + (1440)^2} = 4440$ psi

$\alpha = \arctan \dfrac{1440}{4200} = 18.92°$

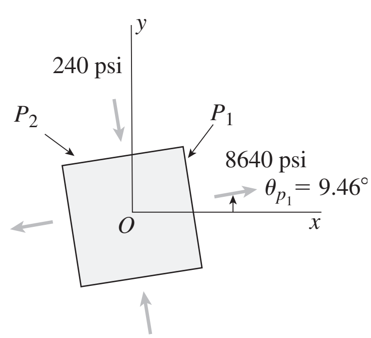
(a) PRINCIPAL STRESSES

$2\theta_{p_1} = \alpha = 18.92° \quad \theta_{p_1} = 9.46°$

$2\theta_{p_2} = 180° + \alpha = 198.92° \quad \theta_{p_2} = 99.46°$

Point P_1: $\sigma_1 = 4200 + R = 8640$ psi

Point P_2: $\sigma_2 = 4200 - R = -240$ psi



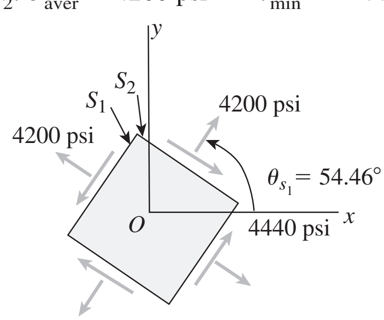
(b) MAXIMUM SHEAR STRESSES

$2\theta_{s_1} = -(90° - \alpha) = -71.08° \quad \theta_{s_1} = -35.54°$

$2\theta_{s_2} = 90° + \alpha = 108.92° \quad \theta_{s_2} = 54.46°$

Point S_1: $\sigma_{aver} = 4200$ psi $\quad \tau_{max} = R = 4440$ psi

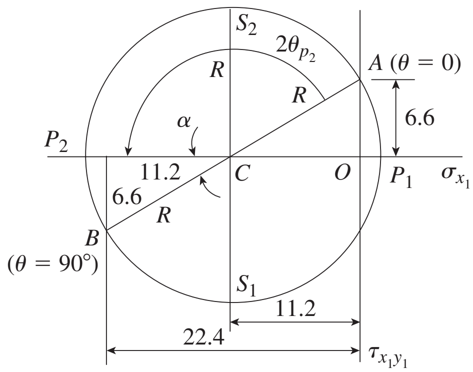
Point S_2: $\sigma_{aver} = 4200$ psi $\quad \tau_{min} = -4440$ psi



Data for 7.4-18 $\sigma_x = 0, \sigma_y = -22.4$ MPa, $\tau_{xy} = -6.6$ MPa

Solution 7.4-18 Principal stresses

$\sigma_x = 0 \quad \sigma_y = -22.4$ MPa
$\tau_{xy} = -6.6$ MPa
(All stresses in MPa)

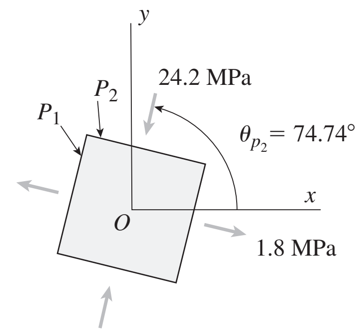


$R = \sqrt{(11.2)^2 + (6.6)^2} = 13.0$ MPa

$\alpha = \arctan \dfrac{6.6}{11.2} = 30.51°$

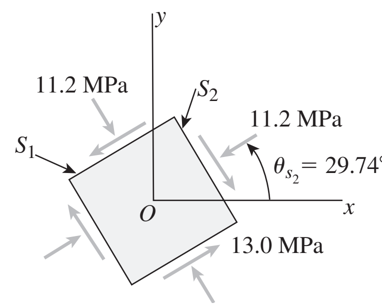
(a) Principal stresses

$2\theta_{p_1} = -\alpha = -30.51° \quad \theta_{p_1} = -15.26°$
$2\theta_{p_2} = 180° - \alpha = 149.49° \quad \theta_{p_2} = 74.74°$
Point P_1: $\sigma_1 = R - 11.2 = 1.8$ MPa
Point P_2: $\sigma_2 = -11.2 - R = -24.2$ MPa



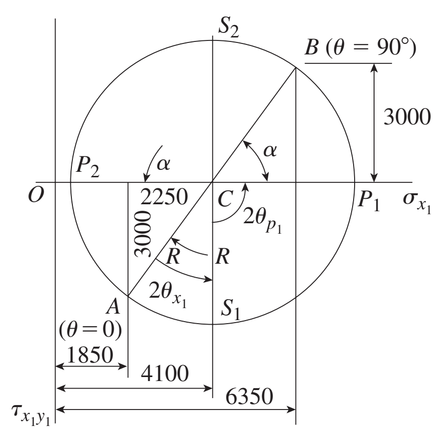
(b) Maximum shear stresses

$2\theta_{s_1} = -\alpha - 90° = -120.51° \quad \theta_{s_1} = -60.26°$
$2\theta_{s_2} = 90° - \alpha = 59.49° \quad \theta_{s_2} = 29.74°$
Point S_1: $\sigma_{aver} = -11.2$ MPa $\quad \tau_{max} = R = 13.0$ MPa
Point S_2: $\sigma_{aver} = -11.2$ MPa $\quad \tau_{min} = -13.0$ MPa



Data for 7.4-19 $\sigma_x = 1850$ psi, $\sigma_y = 6350$ psi, $\tau_{xy} = 3000$ psi

Solution 7.4-19 Principal stresses



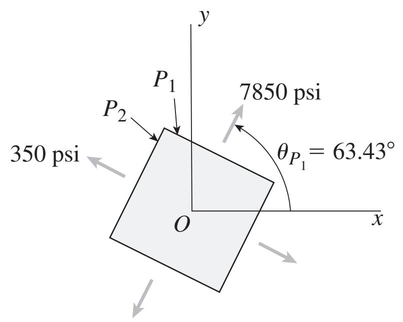
$\sigma_x = 1850$ psi $\quad \sigma_y = 6350$ psi $\quad \tau_{xy} = 3000$ psi
(All stresses in psi)

$R = \sqrt{(2250)^2 + (3000)^2} = 3750$ psi

$\alpha = \arctan \dfrac{3000}{2260} = 53.13°$

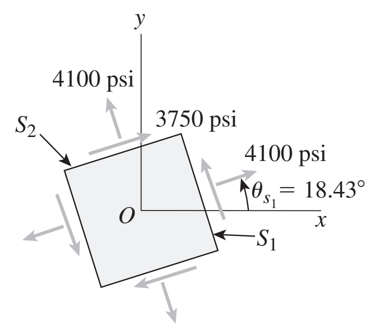
(a) PRINCIPAL STRESSES

$2\theta_{p_1} = 180° - \alpha = 126.87° \quad \theta_{p_1} = 63.43°$
$2\theta_{p_2} = -\alpha = -53.13° \quad \theta_{p_2} = -26.57°$
Point P_1: $\sigma_1 = 4100 + R = 7850$ psi
Point P_2: $\sigma_2 = 4100 - R = 350$ psi



(b) MAXIMUM SHEAR STRESSES

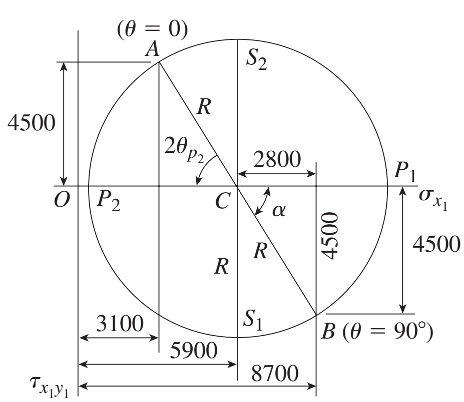
$2\theta_{s_1} = 90° - \alpha = 36.87° \quad \theta_{s_1} = 18.43°$
$2\theta_{s_2} = 270° - \alpha = 216.87° \quad \theta_{s_2} = 108.43°$
Point S_1: $\sigma_{aver} = 4100$ psi $\quad \tau_{max} = R = 3750$ psi
Point S_2: $\sigma_{aver} = 4100$ psi $\quad \tau_{min} = -3750$ psi



Data for 7.4-20 $\sigma_x = 3100$ kPa, $\sigma_y = 8700$ kPa, $\tau_{xy} = -4500$ kPa

Solution 7.4-20 Principal stresses

$\sigma_x = 3100$ kPa $\quad \sigma_y = 8700$ kPa
$\tau_{xy} = -4500$ kPa
(All stresses in kPa)

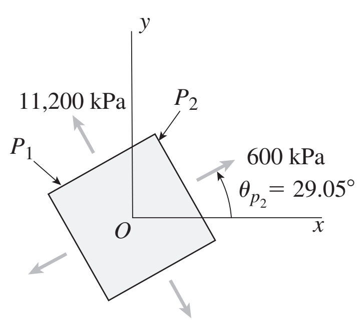


$R = \sqrt{(2800)^2 + (4500)^2} = 5300$ kPa

$\alpha = \arctan \frac{4500}{2800} = 58.11°$

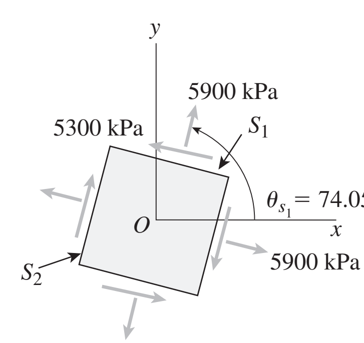
(a) PRINCIPAL STRESSES

$2\theta_{p_1} = \alpha + 180° = 238.11° \quad \theta_{p_1} = 119.05°$
$2\theta_{p_2} = \alpha = 58.11° \quad \theta_{p_2} = 29.05°$
Point P_1: $\sigma_1 = 5900 + R = 11{,}200$ kPa
Point P_2: $\sigma_2 = 5900 - R = 600$ kPa



(b) MAXIMUM SHEAR STRESSES

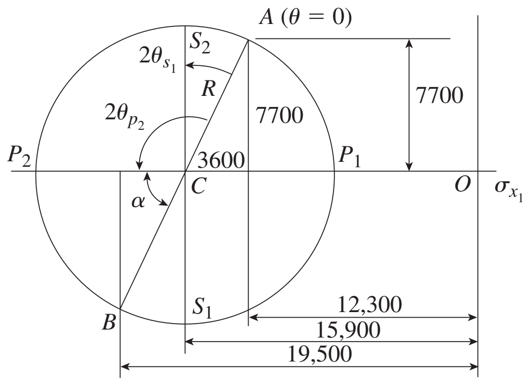
$2\theta_{s_1} = 90° + \alpha = 148.11° \quad \theta_{s_1} = 74.05°$
$2\theta_{s_2} = 270° + \alpha = 328.11° \quad \theta_{s_2} = 164.05°$
Point S_1: $\sigma_{aver} = 5900$ kPa $\quad \tau_{max} = R = 5300$ kPa
Point S_2: $\sigma_{aver} = 5900$ kPa $\quad \tau_{min} = -5300$ kPa



Data for 7.4-21 $\sigma_x = -12{,}300$ psi, $\sigma_y = -19{,}500$ psi, $\tau_{xy} = -7700$ psi

Solution 7.4-21 Principal stresses

$\sigma_x = -12{,}300$ psi $\quad \sigma_y = -19{,}500$ psi
$\tau_{xy} = -7700$ psi
(All stresses in psi)

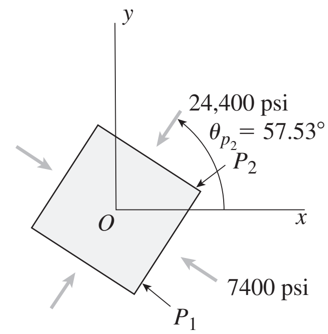


$R = \sqrt{(3600)^2 + (7700)^2} = 8500$ psi

$\alpha = \arctan \dfrac{7700}{3600} = 64.94°$

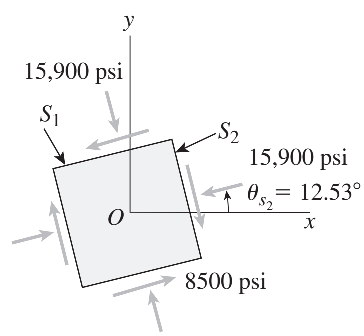
(a) PRINCIPAL STRESSES

$2\theta_{p_1} = -\alpha = -64.94° \quad \theta_{p_1} = -32.47°$
$2\theta_{p_2} = 180° - \alpha = 115.06° \quad \theta_{p_2} = 57.53°$
Point P_1: $\sigma_1 = -15{,}900 + R = -7400$ psi
Point P_2: $\sigma_2 = -15{,}900 - R = -24{,}400$ psi



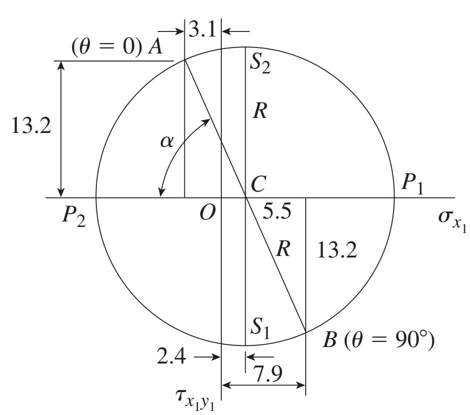
(b) MAXIMUM SHEAR STRESSES

$2\theta_{s_1} = 270° - \alpha = 205.06° \qquad \theta_{s_1} = 102.53°$
$2\theta_{s_2} = 90° - \alpha = 25.06° \qquad \theta_{s_2} = 12.53°$
Point S_1: $\sigma_{aver} = -15{,}900$ psi $\quad \tau_{max} = R = 8500$ psi
Point S_2: $\sigma_{aver} = -15{,}900$ psi $\quad \tau_{min} = -8500$ psi



Data for 7.4-22 $\sigma_x = -3.1$ MPa, $\sigma_y = 7.9$ MPa, $\tau_{xy} = -13.2$ MPa

Solution 7.4-22 Principal stresses



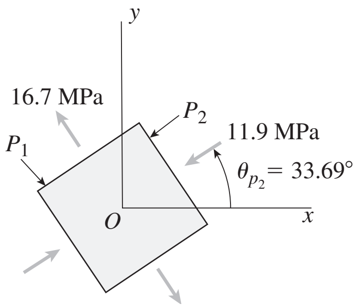
$\sigma_x = -3.1$ MPa $\quad \sigma_y = 7.9$ MPa
$\tau_{xy} = -13.2$ MPa
(All stresses in MPa)

$R = \sqrt{(5.5)^2 + (13.2)^2} = 14.3$ MPa

$\alpha = \arctan \dfrac{13.2}{5.5} = 67.38°$

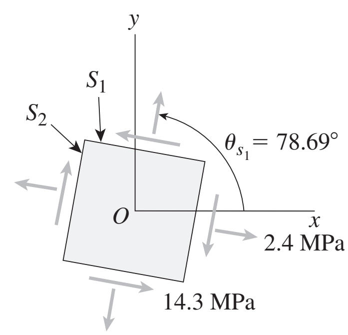
(a) PRINCIPAL STRESSES

$2\theta_{p_1} = 180° + \alpha = 247.38°$ $\quad \theta_{p_1} = 123.69°$
$2\theta_{p_2} = \alpha = 67.38°$ $\quad \theta_{p_2} = 33.69°$
Point P_1: $\sigma_1 = 2.4 + R = 16.7$ MPa
Point P_2: $\sigma_2 = -R + 2.4 = -11.9$ MPa



(b) MAXIMUM SHEAR STRESSES

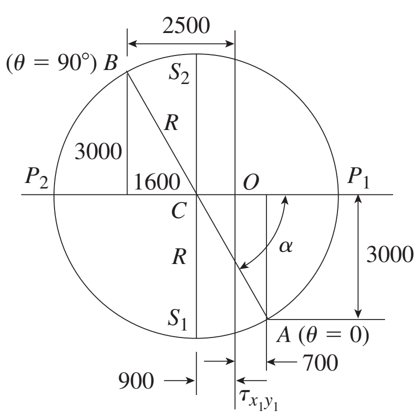
$2\theta_{s_1} = \alpha + 90° = 157.38°$ $\quad \theta_{s_1} = 78.69°$
$2\theta_{s_2} = -90° + \alpha = -22.62°$ $\quad \theta_{s_2} = -11.31°$
Point S_1: $\sigma_{aver} = 2.4$ MPa $\quad \tau_{max} = R = 14.3$ MPa
Point S_2: $\sigma_{aver} = 2.4$ MPa $\quad \tau_{min} = -14.3$ MPa



Data for 7.4-23 $\sigma_x = 700$ psi, $\sigma_y = -2500$ psi, $\tau_{xy} = 3000$ psi

Solution 7.4-23 Principal stresses

$\sigma_x = 700$ psi $\quad \sigma_y = -2500$ psi
$\tau_{xy} = 3000$ psi
(All stresses in psi)

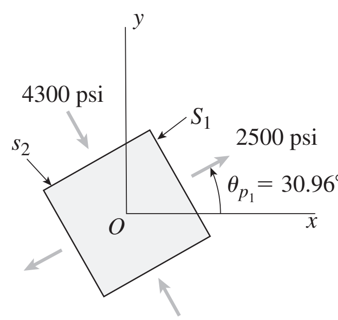


$R = \sqrt{(1600)^2 + (3000)^2} = 3400$ psi

$\alpha = \arctan \frac{3000}{1600} = 61.93°$

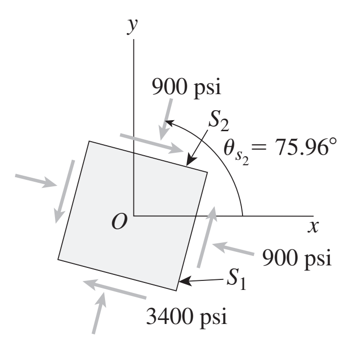
(a) PRINCIPAL STRESSES

$2\theta_{p_1} = \alpha = 61.93°$ $\quad \theta_{p_1} = 30.96°$
$2\theta_{p_2} = 180° + \alpha = 241.93°$ $\quad \theta_{p_2} = 120.96°$
Point P_1: $\sigma_1 = -900 + R = 2500$ psi
Point P_2: $\sigma_2 = -900 - R = -4300$ psi



(b) MAXIMUM SHEAR STRESSES

$2\theta_{s_1} = -90° + \alpha = -28.07°$ $\quad \theta_{s_1} = -14.04°$
$2\theta_{s_2} = 90° + \alpha = 151.93°$ $\quad \theta_{s_2} = 75.96°$
Point S_1: $\sigma_{aver} = -900$ psi $\quad \tau_{max} = R = 3400$ psi
Point S_2: $\sigma_{aver} = -900$ psi $\quad \tau_{min} = -3400$ psi

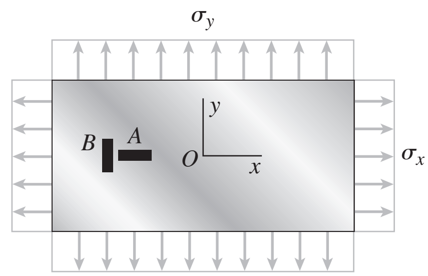


Hooke's Law for Plane Stress

When solving the problems for Section 7.5, assume that the material is linearly elastic with modulus of elasticity E and Poisson's ratio ν.

Problem 7.5-1 A rectangular steel plate with thickness $t = 0.25$ in. is subjected to uniform normal stresses σ_x and σ_y, as shown in the figure. Strain gages A and B, oriented in the x and y directions, respectively, are attached to the plate. The gage readings give normal strains $\epsilon_x = 0.0010$ (elongation) and $\epsilon_y = -0.0007$ (shortening).

Knowing that $E = 30 \times 10^6$ psi and $\nu = 0.3$, determine the stresses σ_x and σ_y and the change Δt in the thickness of the plate.



Probs. 7.5-1 and 7.5-2

Solution 7.5-1 Rectangular plate in biaxial stress

$t = 0.25$ in. $\varepsilon_x = 0.0010$ $\varepsilon_y = -0.0007$
$E = 30 \times 10^6$ psi $\nu = 0.3$
Substitute numerical values:
Eq. (7-40a):

$$\sigma_x = \frac{E}{(1-\nu^2)}(\varepsilon_x + \nu\varepsilon_y) = 26{,}040 \text{ psi} \longleftarrow$$

Eq. (7-40b):

$$\sigma_y = \frac{E}{(1-\nu^2)}(\varepsilon_y + \nu\varepsilon_x) = -13{,}190 \text{ psi} \longleftarrow$$

Eq. (7-39c):

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -128.5 \times 10^{-6}$$

$$\Delta t = \varepsilon_z t = -32.1 \times 10^{-6} \text{ in.} \longleftarrow$$

(Decrease in thickness)

Problem 7.5-2 Solve the preceding problem if the thickness of the steel plate is $t = 10$ mm, the gage readings are $\epsilon_x = 480 \times 10^{-6}$ (elongation) and $\epsilon_y = 130 \times 10^{-6}$ (elongation), the modulus is $E = 200$ GPa, and Poisson's ratio is $\nu = 0.30$.

Solution 7.5-2 Rectangular plate in biaxial stress

$t = 10$ mm $\varepsilon_x = 480 \times 10^{-6}$
$\varepsilon_y = 130 \times 10^{-6}$
$E = 200$ GPa $\nu = 0.3$
Substitute numerical values:
Eq. (7-40a):

$$\sigma_x = \frac{E}{(1-\nu^2)}(\varepsilon_x + \nu\varepsilon_y) = 114.1 \text{ MPa} \longleftarrow$$

Eq. (7-40b):

$$\sigma_y = \frac{E}{(1-\nu^2)}(\varepsilon_y + \nu\varepsilon_x) = 60.2 \text{ MPa} \longleftarrow$$

Eq. (7-39c):

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -261.4 \times 10^{-6}$$

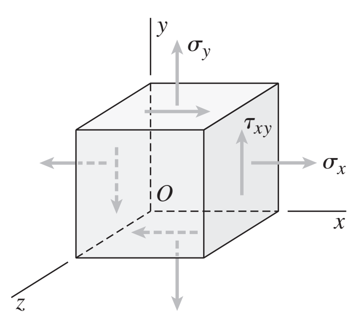
$$\Delta t = \varepsilon_z t = -2610 \times 10^{-6} \text{ mm} \longleftarrow$$

(Decrease in thickness)

Problem 7.5-3 Assume that the normal strains ϵ_x and ϵ_y for an element in *plane stress* (see figure) are measured with strain gages.

(a) Obtain a formula for the normal strain ϵ_z in the z direction in terms of ϵ_x, ϵ_y, and Poisson's ratio ν.

(b) Obtain a formula for the dilatation e in terms of ϵ_x, ϵ_y, and Poisson's ratio ν.



Solution 7.5-3 Plane stress

Given: $\varepsilon_x, \varepsilon_y, \nu$

(a) NORMAL STRAIN ε_z

Eq. (7-34c): $\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y)$

Eq. (7-36a): $\sigma_x = \frac{E}{(1-\nu^2)}(\varepsilon_x + \nu\varepsilon_y)$

Eq. (7-36b): $\sigma_y = \frac{E}{(1-\nu^2)}(\varepsilon_y + \nu\varepsilon_x)$

Substitute σ_x and σ_y into the first equation and simplify:

$$\varepsilon_z = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) \longleftarrow$$

(b) DILATATION

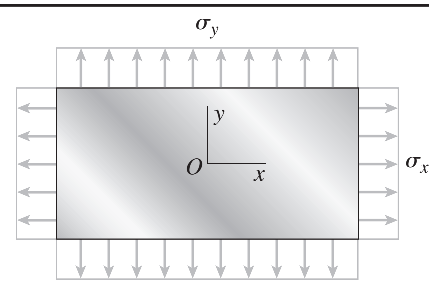
Eq. (7-47): $e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y)$

Substitute σ_x and σ_y from above and simplify:

$$e = \frac{1-2\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) \longleftarrow$$

Problem 7.5-4 A magnesium plate in *biaxial stress* is subjected to tensile stresses $\sigma_x = 24$ MPa and $\sigma_y = 12$ MPa (see figure). The corresponding strains in the plate are $\epsilon_x = 440 \times 10^{-6}$ and $\epsilon_y = 80 \times 10^{-6}$.

Determine Poisson's ratio ν and the modulus of elasticity E for the material.



Probs. 7.5-4 through 7.5-7

Solution 7.5-4 Biaxial stress

$\sigma_x = 24$ MPa $\quad \sigma_y = 12$ MPa

$\varepsilon_x = 440 \times 10^{-6} \quad \varepsilon_y = 80 \times 10^{-6}$

POISSON'S RATIO AND MODULUS OF ELASTICITY

Eq. (7-39a): $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$

Eq. (7-39b): $\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$

Substitute numerical values:

$E(440 \times 10^{-6}) = 24 \text{ MPa} - \nu(12 \text{ MPa})$

$E(80 \times 10^{-6}) = 12 \text{ MPa} - \nu(24 \text{ MPa})$

Solve simultaneously:

$\nu = 0.35 \quad E = 45 \text{ GPa} \longleftarrow$

Problem 7.5-5 Solve the preceding problem for a steel plate with $\sigma_x = 10{,}800$ psi (tension), $\sigma_y = -5400$ psi (compression), $\epsilon_x = 420 \times 10^{-6}$ (elongation), and $\epsilon_y = -300 \times 10^{-6}$ (shortening).

Solution 7.5-5 Biaxial stress

$\sigma_x = 10{,}800$ psi $\quad \sigma_y = -5400$ psi

$\varepsilon_x = 420 \times 10^{-6} \quad \varepsilon_y = -300 \times 10^{-6}$

POISSON'S RATIO AND MODULUS OF ELASTICITY

Eq. (7-39a): $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$

Eq. (7-39b): $\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$

Substitute numerical values:

$E(420 \times 10^{-6}) = 10{,}800 \text{ psi} - \nu(-5400 \text{ psi})$

$E(-300 \times 10^{-6}) = -5400 \text{ psi} - \nu(10{,}800 \text{ psi})$

Solve simultaneously:

$\nu = 1/3 \quad E = 30 \times 10^6 \text{ psi} \longleftarrow$

Problem 7.5-6 A rectangular plate in *biaxial stress* (see figure) is subjected to normal stresses $\sigma_x = 90$ MPa (tension) and $\sigma_y = -20$ MPa (compression). The plate has dimensions $400 \times 800 \times 20$ mm and is made of steel with $E = 200$ GPa and $\nu = 0.30$.

(a) Determine the maximum in-plane shear strain γ_{max} in the plate.
(b) Determine the change Δt in the thickness of the plate.
(c) Determine the change ΔV in the volume of the plate.

Solution 7.5-6 Biaxial stress

$\sigma_x = 90$ MPa $\quad \sigma_y = -20$ MPa
$E = 200$ GPa $\quad \nu = 0.30$
Dimensions of Plate: 400 mm × 800 mm × 20 mm
Shear Modulus (Eq. 7-38):

$$G = \frac{E}{2(1+\nu)} = 76.923 \text{ GPa}$$

(a) MAXIMUM IN-PLANE SHEAR STRAIN

Principal stresses, $\sigma_1 = 90$ MPa $\quad \sigma_2 = -20$ MPa

Eq. (7-26): $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2} = 55.0$ MPa

Eq. (7-35): $\gamma_{max} = \dfrac{\tau_{max}}{G} = 715 \times 10^{-6}$ ←

(b) CHANGE IN THICKNESS

Eq. (7-39c): $\varepsilon_z = -\dfrac{\nu}{E}(\sigma_x + \sigma_y) = -105 \times 10^{-6}$

$\Delta t = \varepsilon_z t = -2100 \times 10^{-6}$ mm ←
(Decrease in thickness)

(c) CHANGE IN VOLUME

From Eq. (7-47): $\Delta V = V_0\left(\dfrac{1-2\nu}{E}\right)(\sigma_x + \sigma_y)$

$V_0 = (400)(800)(20) = 6.4 \times 10^6 \text{ mm}^3$

Also, $\left(\dfrac{1-2\nu}{E}\right)(\sigma_x + \sigma_y) = 140 \times 10^{-6}$

$\therefore \quad \Delta V = (6.4 \times 10^6 \text{ mm}^3)(140 \times 10^{-6})$
$= 896 \text{ mm}^3$ ←
(Increase in volume)

Problem 7.5-7 Solve the preceding problem for an aluminum plate with $\sigma_x = 12{,}000$ psi (tension), $\sigma_y = -3{,}000$ psi (compression), dimensions $20 \times 30 \times 0.5$ in., $E = 10.5 \times 10^6$ psi, and $\nu = 0.33$.

Solution 7.5-7 Biaxial stress

$\sigma_x = 12{,}000$ psi $\quad \sigma_y = -3{,}000$ psi
$E = 10.5 \times 10^6$ psi $\quad \nu = 0.33$
Dimensions of Plate: 20 in. × 30 in. × 0.5 in.
Shear Modulus (Eq. 7-38):

$$G = \frac{E}{2(1+\nu)} = 3.9474 \times 10^6 \text{ psi}$$

(a) MAXIMUM IN-PLANE SHEAR STRAIN

Principal stresses: $\sigma_1 = 12{,}000$ psi
$\sigma_2 = -3{,}000$ psi

Eq. (7-26): $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2} = 7{,}500$ psi

Eq. (7-35): $\gamma_{max} = \dfrac{\tau_{max}}{G} = 1{,}900 \times 10^{-6}$ ←

(b) CHANGE IN THICKNESS

Eq. (7-39c): $\varepsilon_z = -\dfrac{\nu}{E}(\sigma_x + \sigma_y) = -282.9 \times 10^{-6}$

$\Delta t = \varepsilon_z t = -141 \times 10^{-6}$ in. ←
(Decrease in thickness)

(c) CHANGE IN VOLUME

From Eq. (7-47): $\Delta V = V_0\left(\dfrac{1-2\nu}{E}\right)(\sigma_x + \sigma_y)$

$V_0 = (20)(30)(0.5) = 300 \text{ in.}^3$

Also, $\left(\dfrac{1-2\nu}{E}\right)(\sigma_x + \sigma_y) = 291.4 \times 10^{-6}$

$\therefore \quad \Delta V = (300 \text{ in.}^3)(291.4 \times 10^{-6})$
$= 0.0874 \text{ in.}^3$ ←
(Increase in volume)